

NEUTRAL KAONS IN RANDOM MEDIA

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Abstract

The generalized dynamics describing the propagation of neutral kaons in randomly fluctuating media is derived and analyzed. It takes into account possible matter-induced effects leading to loss of phase coherence and dissipation. The study of selected neutral kaon observables indicates that these non-standard effects are amenable to a direct experimental analysis.

1. INTRODUCTION

When a quantum system is immersed in an external environment, its time evolution is no longer unitary, since in general it exchanges energy and entropy with the larger system. In principle, the subdynamics can be readily obtained from the total, unitary system+environment evolution by eliminating (*i.e.* integrating over) the degrees of freedom pertaining to the environment. In practice however, this procedure leads to tractable dynamical equations only when the interaction between subsystem and environment can be considered weak: in these cases memory effects disappear, and the time evolution of the subsystem is described by linear maps, encoding non-hamiltonian physical effects, like irreversibility and dissipation.[1-4]

The system plus environment paradigm for the treatment of the so-called open systems is nevertheless very general and has been successfully adopted to model very different physical situations, in laser, atomic and molecular physics.[1-8] In particular, it has been very useful in describing the effects of random media or of stochastic external fields in particle propagation inside interferometric devices.[9-13] Indeed, for weakly-coupled systems, the decoherence and dissipative phenomena induced by the media are in general very small, so that the most suited way to study them is through appropriate interferometric set-ups.[14-16]

Motivated by these results, in the following similar techniques will be adopted to study effects of irreversibility and dissipation in neutral kaons propagating inside a random medium. Because of strangeness oscillations, neutral kaons have been one of the prime laboratory for the discovery and analysis of small physical effects, CP violation being the most striking example;[17,18] therefore, they appear to be the natural place for studying environment-induced dissipative effects, also in view of the unique opportunities offered by the production of correlated kaons at ϕ -factories.[19]

Matter effects in the physics of neutral kaons have been studied since the very early days of kaon physics. However, most of the attention has been devoted to the so called regeneration phenomena, where short-lived neutral kaon states are regenerated in a beam of purely long-lived ones via the passage through a thin slab of material: this is due to the coherent interactions of the kaons with the nuclei of the material, that produce different scattering amplitudes for the kaon and antikaon components of the impinging beam. One can show that this effect is dominated by forward scattering processes, and it can be described by an effective hamiltonian involving the index of refraction of the material.[20-23]

Here, we shall generalize this physical situation by considering the propagation of neutral kaons in a randomly fluctuating medium. The kaon system can then be viewed as an open system, where the environment (the medium) is described by a classical, random external field.

Quite in general, any environment can be modelled in this way, provided the characteristic decay time of the associated correlations is sufficiently small with respect to the typical evolution time τ_S of the subsystem. In the case of the kaons, τ_S can be roughly identified with the lifetime of the short-lived kaon, so that the correlations in the material through which the kaons propagate must decay very rapidly, or equivalently, the medium must fluctuate on times much shorter than τ_S . Although this condition looks quite restric-

tive, it can be met quite easily by a careful choice of material. Indeed, many short-time physical phenomena, like molecular vibrational motion and relaxation, or collisions in liquids, take place at times that are at least a couple of orders of magnitude smaller than τ_S .¹⁾ Therefore, a kaon moving *e.g.* in a gas at sufficiently high temperature or in a liquid would see a random fluctuating distribution of centers of scattering, and therefore be subjected to stochastic incoherent interactions with the material; as a consequence, its dynamics can be effectively modelled as being that of a particle propagating in a random medium.

This physical situation is clearly different from the one encountered in the regeneration phenomena, where a single coherent scattering in the thin slab is enough to account for the effect. In the present case instead, correlations in the material play a fundamental role: as we shall see, they are responsible for the generation of irreversibility and loss of quantum coherence.²⁾

In the next section, we shall discuss in detail the derivation of the master equation that describes within the weak-coupling hypothesis the propagation of neutral kaons in random media. Since it incorporates the presence of dissipative phenomena, it can not be fully written in the familiar hamiltonian form; rather, it assumes the structure appropriate for generating a quantum dynamical semigroup.[1-4] Specific examples are presented in Sect.3, while some general properties of its solutions are discussed in Sect.4. Since the matter induced effects are small, a suitable perturbative approximation can be used, allowing the explicit evaluation of relevant kaon observables: Sect.5 and 6 will be devoted to the analysis of the behaviour of these observables. We shall first study the decays of single kaons, relevant for fixed-target experiments, and then discuss the case of correlated kaons at ϕ -factories. In both cases, the matter-induced phenomena modify in a very specific way the various kaon observables, that are therefore amenable to a direct experimental analysis. In particular as discussed in the final Sect.7, at ϕ -factories one can adjust the experimental conditions so as to allow the propagation in the medium of only one of the two correlated kaons; in this configuration, one can easily perform tests on the physical consistency of the adopted treatment, shedding further light on the general description of open quantum systems.

Although preliminary, we hope that the results presented in our investigation will stimulate further studies on the dynamics of neutral kaons in random media, encouraging in particular a detailed experimental analysis.

¹⁾ The direct study of these very short-lived phenomena have been made possible by the recent introduction of lasers that are able to produce pulses reaching the femtosecond scale; for a review, see [24].

²⁾ Using a more phenomenological approach, non-standard effects in the propagation of neutral kaons in a stationary, low-density material have been recently discussed in [25].

2. MASTER EQUATION

We shall work within the familiar effective description of the neutral kaon system which requires the introduction of a two-dimensional Hilbert space; [17-19] the set $\{|K^0\rangle, |\overline{K}^0\rangle\}$ constitutes a convenient basis in this space. With respect to this basis, single kaon states can then be represented by density matrices, \widehat{R} , *i.e.* by hermitian 2×2 matrices with non-negative eigenvalues. Their time evolution is described in terms of an equation of a standard Liouville-von Neumann form:

$$\frac{\partial \widehat{R}(t)}{\partial t} = -iH^{(0)} \widehat{R}(t) + i\widehat{R}(t) H^{(0)\dagger} + L_t[\widehat{R}(t)]. \quad (2.1)$$

The first two pieces on the r.h.s. give the usual hamiltonian contribution, while the additional linear map L_t , explicitly given in (2.10) below, takes into account the presence of the stochastic medium.

The effective hamiltonian $H^{(0)}$, the so-called Weisskopf-Wigner hamiltonian, describes the propagation of the kaons in vacuum;³⁾ it contains a non-hermitian term, characterized by the natural width of the physical states. The entries of $H^{(0)}$ can be expressed in terms of its complex eigenvalues, $\lambda_S^{(0)} = m_S^{(0)} - i\gamma_S^{(0)}/2$, $\lambda_L^{(0)} = m_L^{(0)} - i\gamma_L^{(0)}/2$, and the complex parameters $p_S^{(0)}$, $q_S^{(0)}$, $p_L^{(0)}$, $q_L^{(0)}$, appearing in the corresponding (right) eigenstates,

$$\begin{aligned} |K_S\rangle &= p_S^{(0)} |K^0\rangle + q_S^{(0)} |\overline{K}^0\rangle, & |p_S^{(0)}|^2 + |q_S^{(0)}|^2 &= 1, \\ |K_L\rangle &= p_L^{(0)} |K^0\rangle - q_L^{(0)} |\overline{K}^0\rangle, & |p_L^{(0)}|^2 + |q_L^{(0)}|^2 &= 1. \end{aligned} \quad (2.2)$$

For later convenience, it is useful to introduce the following positive combinations, involving the eigenstates masses and widths,

$$\Delta m^{(0)} = m_L^{(0)} - m_S^{(0)}, \quad \Delta \Gamma^{(0)} = \gamma_S^{(0)} - \gamma_L^{(0)}, \quad \Gamma^{(0)} = \frac{\gamma_S^{(0)} + \gamma_L^{(0)}}{2}, \quad (2.3)$$

as well as the complex quantities:

$$\Gamma_{\pm}^{(0)} = \Gamma^{(0)} \pm i\Delta m^{(0)}, \quad \Delta \Gamma_{\pm}^{(0)} = \Delta \Gamma^{(0)} \pm 2i\Delta m^{(0)}. \quad (2.4)$$

The effective Hamiltonian $H^{(0)}$ can be diagonalized using the similarity transformation induced by (2.2):

$$H^{(0)} = V^{(0)} H_D^{(0)} V^{(0)-1}, \quad (2.5)$$

with

$$V^{(0)} = \begin{bmatrix} p_S^{(0)} & p_L^{(0)} \\ q_S^{(0)} & -q_L^{(0)} \end{bmatrix}, \quad H_D^{(0)} = \begin{bmatrix} \lambda_S^{(0)} & 0 \\ 0 & \lambda_L^{(0)} \end{bmatrix}. \quad (2.6)$$

³⁾ Throughout the paper, we shall append a superscript ⁽⁰⁾ to all quantities referring to neutral kaons in vacuum, *i.e.* in absence of the medium.

Then, one can write:

$$H^{(0)} = \frac{\lambda_S^{(0)} + \lambda_L^{(0)}}{2} + \frac{\lambda_S^{(0)} - \lambda_L^{(0)}}{2} \begin{bmatrix} \theta^{(0)} & \frac{2\sigma^{(0)}}{r_S^{(0)} + r_L^{(0)}} \\ \frac{2}{r_S^{(0)} + r_L^{(0)}} & -\theta^{(0)} \end{bmatrix} ; \quad (2.7)$$

the two complex parameters

$$\theta^{(0)} = \frac{r_S^{(0)} - r_L^{(0)}}{r_S^{(0)} + r_L^{(0)}} , \quad \sigma^{(0)} = r_S^{(0)} r_L^{(0)} , \quad (2.8)$$

involving the ratios,

$$r_S^{(0)} = \frac{p_S^{(0)}}{q_S^{(0)}} , \quad r_L^{(0)} = \frac{p_L^{(0)}}{q_L^{(0)}} , \quad (2.9)$$

signal CPT (for $\theta^{(0)} \neq 0$) and T (for $\xi^{(0)} \equiv (|\sigma^{(0)}| - 1)/(|\sigma^{(0)}| + 1) \neq 0$) violating effects in mixing, respectively.

All this is valid in vacuum; when the neutral kaons propagate in matter, the interaction with the nuclei of the material gives additional contributions to the evolution equation, the term L_t in (2.1). As explained in the Introduction, we shall consider the case of rapidly fluctuating media, that can be represented by classical stochastic fields. The action of the media on the kaons can then be expressed in generalized hamiltonian form:

$$L_t[\hat{R}(t)] = -iF(t)\hat{R}(t) + i\hat{R}(t)[F(t)]^\dagger , \quad (2.10)$$

where

$$F(t) = \sum_{\mu=0}^3 F_\mu(t) \sigma_\mu , \quad (2.11)$$

while σ_0 is the 2×2 unit matrix and $\sigma_1, \sigma_2, \sigma_3$ the Pauli matrices. The quantities $F_\mu(t)$, $\mu = 0, 1, 2, 3$, generate a complex, Gaussian stochastic field; they are assumed to have in general non-zero mean, but translationally invariant correlations (a star means complex conjugation):

$$\hat{G}_{\mu\nu}(t-s) \equiv \langle F_\mu(t) F_\nu(s) \rangle - \langle F_\mu(t) \rangle \langle F_\nu(s) \rangle , \quad (2.12a)$$

$$\hat{W}_{\mu\nu}(t-s) \equiv \langle F_\mu(t) F_\nu^*(s) \rangle - \langle F_\mu(t) \rangle \langle F_\nu^*(s) \rangle . \quad (2.12b)$$

Since the generalized hamiltonian $F(t)$ in (2.11) involves stochastic variables, the density matrix $\hat{R}(t)$, solution of the total equation of motion (2.1), is also stochastic. Instead, we are interested in the behaviour of the reduced density matrix $\hat{\rho}(t) \equiv \langle \hat{R}(t) \rangle$ which is obtained by averaging over the noise; it is $\hat{\rho}(t)$ that describes the effective evolution of the kaon states in the medium and allows computing the behaviour of relevant observables. We shall now explicitly describe the derivation of an effective master equation for $\hat{\rho}(t)$,

making the additional assumption that kaons and noise be decouple at $t = 0$, so that the initial state is $\hat{\rho}(0) \equiv \langle \hat{R}(0) \rangle = \hat{R}(0)$.⁴⁾

Since the hamiltonian $H^{(0)}$ is statistically independent from the stochastic variables, one can choose to average over the noise in the interaction representation, where we set:

$$\tilde{R}(t) = e^{it H^{(0)}} \hat{R}(t) e^{-it H^{(0)\dagger}} , \quad (2.13)$$

so that:

$$\frac{\partial \tilde{R}(t)}{\partial t} = \tilde{L}_t[\tilde{R}(t)] \equiv -i \sum_{\mu=0}^3 F_{\mu}(t) \sigma_{\mu}(t) \tilde{R}(t) + i \tilde{R}(t) \sum_{\mu=0}^3 F_{\mu}^*(t) [\sigma_{\mu}(t)]^{\dagger} , \quad (2.14)$$

with $\sigma_{\mu}(t) = e^{it H^{(0)}} \sigma_{\mu} e^{-it H^{(0)}}$. The time evolution of the reduced density matrix in the interaction representation, $\tilde{\rho}(t) \equiv \langle \tilde{R}(t) \rangle$, can then be expressed as a series expansion involving multiple correlations of the operator \tilde{L}_t :

$$\tilde{\rho}(t) = M_t[\tilde{\rho}(0)] \equiv \sum_{k=0}^{\infty} M_k[\tilde{\rho}(0)] , \quad (2.15)$$

$$M_k[\tilde{\rho}] = \int_0^t ds_1 \int_0^{s_1} ds_2 \cdots \int_0^{s_{k-1}} ds_k \langle \tilde{L}_{s_1} \tilde{L}_{s_2} \cdots \tilde{L}_{s_k} \rangle [\tilde{\rho}] . \quad (2.16)$$

The sum M_t in (2.15) can be formally inverted, and a suitable resummation gives (a dot represents time derivative):[26]

$$\frac{\partial \tilde{\rho}(t)}{\partial t} = \dot{M}_t M_t^{-1}[\tilde{\rho}(t)] = \left\{ \dot{M}_1 + (\dot{M}_2 - \dot{M}_1 M_1) + \dots \right\} [\tilde{\rho}(t)] . \quad (2.17)$$

As mentioned before and discussed in more detail at the end of the section, the action of the medium on the travelling kaons is weak. Therefore, one can focus the attention on the dominant terms of the previous expansions, neglecting all contributions higher than the second-order ones. Further, since the characteristic decay time of correlations in the medium is by assumption much smaller than the typical time scale of the system, the memory effects implicit in (2.17) should not be physically relevant and the use of the Markovian approximation justified. This is implemented in practice by extending to infinity the upper limit of the integrals appearing in \dot{M}_2 and M_1 (compare with (2.16)).[1-4]

By returning to the Schrödinger representation, one finally obtains the master equation generating the time evolution of the reduced density matrix $\hat{\rho}(t) \equiv \langle \hat{R}(t) \rangle$. It takes the following explicit form:

$$\frac{\partial \hat{\rho}(t)}{\partial t} = -iH \hat{\rho}(t) + i\hat{\rho}(t) H^{\dagger} + \hat{L}[\hat{\rho}(t)] , \quad (2.18a)$$

⁴⁾ This condition is always satisfied in a typical experimental situation, where the kaons enter the medium after being produced.

where

$$H = H^{(0)} + H^{(1)} + H^{(2)} , \quad (2.18b)$$

$$\widehat{L}[\hat{\rho}] = \frac{1}{2} \sum_{i,j=1}^3 \widehat{\mathcal{C}}_{ij} \left[2\sigma_i \hat{\rho} \sigma_j - \{\sigma_j \sigma_i, \hat{\rho}\} \right] . \quad (2.18c)$$

The effective hamiltonian in matter H differs from the one in vacuum $H^{(0)}$ by first order terms (coming from the piece \dot{M}_1 in (2.17)) involving the noise mean values:

$$H^{(1)} = \sum_{\mu=0}^3 \langle F_{\mu}(t) \rangle \sigma_{\mu} , \quad (2.19)$$

and by second-order contributions (coming from the second-order terms in (2.17)) involving the noise correlations (2.12) through the time-independent combinations:

$$\widehat{\mathcal{B}}_{\mu\nu} = \sum_{\lambda=0}^3 \int_0^{\infty} dt \widehat{G}_{\mu\lambda}(t) \mathcal{U}_{\lambda\nu}(-t) , \quad (2.20a)$$

$$\widehat{\mathcal{C}}_{\mu\nu} = \sum_{\lambda=0}^3 \int_0^{\infty} dt \left[\widehat{W}_{\mu\lambda}(t) \mathcal{U}_{\lambda\nu}^*(-t) + \widehat{W}_{\nu\lambda}^*(t) \mathcal{U}_{\lambda\mu}(-t) \right] , \quad (2.20b)$$

where the 4×4 matrix $\mathcal{U}(t)$ is defined by the following transformation rule:

$$\sigma_{\mu}(t) = e^{it H^{(0)}} \sigma_{\mu} e^{-it H^{(0)}} = \mathcal{U}_{\mu\nu}(t) \sigma_{\nu} . \quad (2.21)$$

Explicitly, one finds:

$$\begin{aligned} H^{(2)} = & \frac{i}{2} \sum_{\mu=0}^3 \left(\widehat{\mathcal{C}}_{\mu\mu} - 2\widehat{\mathcal{B}}_{\mu\mu} \right) \sigma_0 \\ & + \sum_{i=1}^3 \left[i \widehat{\mathcal{C}}_{i0} - i \left(\widehat{\mathcal{B}}_{i0} + \widehat{\mathcal{B}}_{0i} \right) + \frac{1}{2} \sum_{j,k=1}^3 \epsilon_{ijk} \left(2\widehat{\mathcal{B}}_{jk} - i \mathcal{I}m \widehat{\mathcal{C}}_{jk} \right) \right] \sigma_i . \end{aligned} \quad (2.22)$$

On the other hand, the additional piece $\widehat{L}[\hat{\rho}]$ in (2.18c) is a time-independent, trace-preserving linear map involving the hermitian 3×3 submatrix $\widehat{\mathcal{C}}_{ij}$, obtained from the coefficient matrix (2.20b) by letting $\mu, \nu = 1, 2, 3$. It introduces irreversibility, inducing in general dissipation and loss of quantum coherence. Altogether, equation (2.18) generates a semigroup of linear maps, $\Gamma_t : \hat{\rho}(0) \mapsto \hat{\rho}(t) \equiv \Gamma_t[\hat{\rho}(0)]$, for which composition is defined only forward in time: $\Gamma_t \circ \Gamma_s = \Gamma_{t+s}$, with $t, s \geq 0$; it is usually referred to as a quantum dynamical semigroup.[1-4]

As a further remark, notice that when correlations in the medium are negligible, *i.e.* the combination in (2.12) are vanishingly small, the physical situation described by

equation (2.18) corresponds to that of the regeneration phenomenon. Indeed, in this case the presence of matter is signaled solely by the shift $H^{(1)}$ in the effective hamiltonian. For a kaon-medium interaction dominated by coherent forward scattering, $H^{(1)}$ becomes diagonal and its components turn out to be expressible in terms of the forward scattering amplitudes $f_K, f_{\bar{K}}$ in the medium:[20-23]

$$H^{(1)} = -\frac{2\pi\nu}{m_K} \begin{bmatrix} f_K & 0 \\ 0 & f_{\bar{K}} \end{bmatrix}, \quad (2.23)$$

where m_K is the K^0 mass, while ν represents the density of scattering centers in the medium.

The amplitudes $f_K, f_{\bar{K}}$ have been directly measured in experiments for many types of materials,[27, 28] and quite consistently one finds: $|H^{(1)}| \simeq 10^{-2} \Delta\Gamma^{(0)}$, where $\Delta\Gamma^{(0)} \simeq \gamma_S^{(0)}$ is the typical inverse timescale describing the kaon evolution in vacuum, as generated by the hamiltonian $H^{(0)}$. In view of the fact that $H^{(1)}$ coincides with the mean value of the stochastic hamiltonian in (2.11), one concludes that also the averages $\langle F_\mu \rangle$ should be in modulus of the same order of magnitude. Noise correlations are however much smaller: from the definition (2.12), one can safely deduce the following rough estimate: $|\widehat{G}| \simeq |\widehat{W}| \simeq |\langle F_\mu \rangle|^2 \simeq 10^{-4} (\Delta\Gamma^{(0)})^2$. Therefore, the weak coupling hypothesis adopted in deriving the master equation (2.18) appears physically justified.

3. EXAMPLES

Before analyzing in detail the evolution in time of the density matrix $\hat{\rho}(t)$, we shall discuss some physically interesting instances of the master equation (2.18); they correspond to specific realizations of the medium through which the kaons propagate.

Some general considerations apply to all cases. As discussed at the end of the previous section, the noise contributions to (2.18) are expected to be small, in particular those involving noise correlations. Therefore, whenever the correlations in (2.12) are multiplied by other small parameters, *e.g.* those coming from the hamiltonian $H^{(0)}$ in (2.7), one can safely neglect them in comparison with the dominant pieces. These considerations are particularly relevant in the evaluation of $\widehat{\mathcal{B}}_{\mu\nu}$ and $\widehat{\mathcal{C}}_{\mu\nu}$ in (2.20). These quantities are linear in the noise correlations: the entries of the matrix $\mathcal{U}(t)$ appearing in (2.20) can then be computed at lowest order. Using the diagonalization (2.5) to write $e^{itH^{(0)}} = V^{(0)} e^{itH_D^{(0)}} V^{(0)-1}$ and further setting $p_S^{(0)} = q_S^{(0)} = p_L^{(0)} = q_L^{(0)} = 1/\sqrt{2}$ in (2.6),⁵⁾ from (2.21) one explicitly finds:

$$\mathcal{U}(t) = \begin{bmatrix} \sigma_0 & 0 \\ 0 & \sigma(t) \end{bmatrix}, \quad \sigma(t) = \begin{bmatrix} \cos \omega t & \sin \omega t \\ -\sin \omega t & \cos \omega t \end{bmatrix}, \quad (3.1)$$

⁵⁾ These conditions, that come from the assumption of *CPT* and *T* (hence *CP*) conservation in mixing, imply a specific phase choice in the definition of the basis state vectors in the Hilbert space; in practice, this poses no problems, since physical observables, being the result of a trace operation (see later), are manifestly phase-invariant.

where $\omega = i\Delta\Gamma_-^{(0)}/2$.

Further, the stochastic medium fluctuates on time intervals much shorter than the typical kaon evolution timescale $1/\Delta\Gamma^{(0)}$. Correspondingly, the noise correlations in (2.12) can be taken to have an exponentially decreasing form, with decay parameter λ much larger than $\Delta\Gamma^{(0)}$. This allows neglecting all higher-order contributions in $\Delta\Gamma^{(0)}/\lambda$ while evaluating explicitly the coefficients $\hat{\mathcal{B}}_{\mu\nu}$ and $\hat{\mathcal{C}}_{\mu\nu}$.

3.1 Generalized regeneration

In this case, the stochastic hamiltonian in (2.11) is taken to be diagonal, so that only the components with $\mu = 0, 3$ of the stochastic variables $F_\mu(t)$ are non vanishing:

$$F(t) = \begin{bmatrix} F_+(t) & 0 \\ 0 & F_-(t) \end{bmatrix}, \quad F_\pm(t) = \frac{1}{2}(F_0(t) \pm F_3(t)). \quad (3.2)$$

Further, we assume the correlation functions to have the form ($\mu, \nu = 0, 3$):

$$\hat{G}_{\mu\nu}(t-s) = G_{\mu\nu} e^{-\lambda_1|t-s|}, \quad (3.3a)$$

$$\hat{W}_{\mu\nu}(t-s) = W_{\mu\nu} e^{-\lambda_2|t-s|}, \quad (3.3b)$$

where $G_{\mu\nu}$ and $W_{\mu\nu}$ are time-independent complex matrices, with $W = W^\dagger$. Physically speaking, this case corresponds to a generalization of the familiar regeneration situation. The kaons are still mainly forward scattered by the medium; however, while travelling in it, they encounter density fluctuations, whose correlations have the general behaviour (3.3).

With the help of (3.1) and (3.3), one can now explicitly compute the noise contributions in (2.18). To lowest order in the small parameters, one finds that only the $i = j = 3$ entry of the matrix $\hat{\mathcal{C}}_{ij}$ is non vanishing,

$$\hat{\mathcal{C}}_{33} = \frac{2W_{33}}{\lambda_2} \geq 0, \quad (3.4)$$

while the hamiltonian contributions $H^{(1)}$ and $H^{(2)}$ turns out to be diagonal:

$$\begin{aligned} H - H^{(0)} &= \left[\langle F_0 \rangle + i \frac{W_{00} + W_{33}}{\lambda_2} - i \frac{G_{00} + G_{33}}{\lambda_1} \right] \sigma_0 + \left[\langle F_3 \rangle + \frac{2iW_{03}^*}{\lambda_2} - \frac{2iG_{03}}{\lambda_1} \right] \sigma_3 \\ &\equiv - \frac{2\pi\nu}{m_K} \begin{bmatrix} f_K & 0 \\ 0 & f_{\bar{K}} \end{bmatrix}. \end{aligned} \quad (3.5)$$

The complex parameters f_K and $f_{\bar{K}}$ now contain both the standard forward scattering amplitudes, described by $\langle F_0 \rangle$ and $\langle F_3 \rangle$, and terms originating from the correlations (3.3).

The total effective hamiltonian H can be expressed as in (2.7) in terms of its eigenvalues λ_S, λ_L and the quantities r_S, r_L, θ, σ in matter, defined through its eigenvectors as

in (2.2), (2.8) and (2.9). The differences between these parameters and the corresponding ones in vacuum can be expressed as a power series expansion in the combination:

$$\eta = \frac{\pi\nu}{m_K} \left[\frac{f_K - f_{\overline{K}}}{\lambda_S^{(0)} - \lambda_L^{(0)}} \right] ; \quad (3.6)$$

for many materials, one finds that $|\eta|$ is of order $10^{-2} \div 10^{-3}$. [27, 28] Although $\sigma = \sigma^{(0)}$ and therefore T violation is unaffected by the medium, to lowest order one obtains: $\theta = \theta^{(0)} - \eta$, so that, not surprisingly, CPT invariance is effectively broken by matter effects even if it is preserved in vacuum; in addition, one has: $r_S = r_S^{(0)} - \eta$, $r_L = r_L^{(0)} + \eta$.

3.2 Diagonal correlations

The noise hamiltonian $F(t)$ in (2.11) is no longer diagonal; in this case the kaons undergo incoherent scatterings not exclusively in the forward direction. However, we assume that the only non vanishing correlation functions in the medium be diagonal, and precisely:

$$\begin{aligned} \widehat{W}_{11}(t-s) &= \widehat{W}_{00}(t-s) = W_{11} e^{-\lambda_1|t-s|} , \\ \widehat{W}_{22}(t-s) &= \widehat{W}_{33}(t-s) = W_{22} e^{-\lambda_2|t-s|} , \\ \widehat{G}_{\mu\mu}(t-s) &= G_{\mu\mu} e^{-\lambda_3|t-s|} , \quad \mu = 0, 1, 2, 3 . \end{aligned} \quad (3.7)$$

The noise contribution to the effective hamiltonian takes now the form:

$$H - H^{(0)} = \left[\langle F_0 \rangle + 2i \left(\frac{W_{11}}{\lambda_1} + \frac{W_{22}}{\lambda_2} \right) - i \sum_{\mu=0}^3 \frac{G_{\mu\mu}}{\lambda_3} \right] \sigma_0 + \sum_{i=1}^3 \langle F_i \rangle \sigma_i , \quad (3.8)$$

while the matrix $\widehat{\mathcal{C}}_{ij}$ which characterizes the dissipative term in (2.18) becomes diagonal:

$$\widehat{\mathcal{C}}_{ij} = \begin{bmatrix} \frac{2W_{11}}{\lambda_1} & & \\ & \frac{2W_{22}}{\lambda_2} & \\ & & \frac{2W_{22}}{\lambda_2} \end{bmatrix} . \quad (3.9)$$

3.3 General case

In general however, the fluctuations in the medium are randomly directed so that the correlation functions in (2.12) assume the generic form (3.3), with all components of the constant matrices $G_{\mu\nu}$ and $W_{\mu\nu}$ non vanishing. In this case, keeping again only dominant contributions, one finds:

$$\widehat{\mathcal{B}}_{\mu\nu} = \frac{G_{\mu\nu}}{\lambda_1} , \quad \widehat{\mathcal{C}}_{\mu\nu} = \frac{2W_{\mu\nu}}{\lambda_2} . \quad (3.10)$$

As clear from the definitions (2.12b) and (3.3), the matrix W turns out to be hermitian and also positive. The same properties clearly hold for the 3×3 coefficient submatrix $\widehat{\mathcal{C}}_{ij}$ characterizing the dissipative contribution in (2.18) as given in (3.10), as well as for the previously discussed cases (3.4) and (3.9). This makes the semigroup $\Gamma_t : \hat{\rho}(0) \mapsto \hat{\rho}(t) = \Gamma_t[\hat{\rho}(0)]$

generated by the master equation (2.18) completely positive. This property is crucial in assuring the consistency of the generalized dynamics Γ_t in all possible situations; [1-3, 15, 29] as we shall see in discussing correlated neutral kaons, lacking of it may lead to physically unacceptable results. It is therefore reassuring that in our treatment the property of complete positivity emerges naturally, without further assumptions.⁶⁾

Although the matrix $\widehat{\mathcal{C}}_{ij}$ turns out to be in general complex, in the following we shall limit our considerations only to situations for which $\widehat{\mathcal{C}}_{ij}$ results real. These correspond to the physically most interesting cases; in fact, the reality condition guarantees the increase of the von Neumann entropy, $dS/dt \geq 0$, $S = -\text{Tr}(\hat{\rho} \ln \hat{\rho})$, [29] which is a physically desirable requirement for the dynamics of a small system, the neutral kaon, in weak interaction with a large environment, the medium. For later convenience, we shall parametrize the entries of $\widehat{\mathcal{C}}_{ij}$ using the real constants

$$\begin{aligned} a &= \widehat{\mathcal{C}}_{11} + \widehat{\mathcal{C}}_{22} & b &= \widehat{\mathcal{C}}_{23} \\ \alpha &= \widehat{\mathcal{C}}_{11} + \widehat{\mathcal{C}}_{33} & c &= -\widehat{\mathcal{C}}_{13} \\ \gamma &= \widehat{\mathcal{C}}_{22} + \widehat{\mathcal{C}}_{33} & \beta &= \widehat{\mathcal{C}}_{12} . \end{aligned} \quad (3.11)$$

The positivity condition on $\widehat{\mathcal{C}}_{ij}$ then requires:

$$\begin{aligned} 2R &\equiv \alpha + \gamma - a \geq 0 , & RS - b^2 &\geq 0 , \\ 2S &\equiv a + \gamma - \alpha \geq 0 , & RT - c^2 &\geq 0 , \\ 2T &\equiv a + \alpha - \gamma \geq 0 , & ST - \beta^2 &\geq 0 , \\ RST - 2bc\beta - R\beta^2 - Sc^2 - Tb^2 &\geq 0 . \end{aligned} \quad (3.12)$$

3.4 White noise

Finally, let us consider the case of a medium that behaves as a white noise; it is described by stochastic variables $F_\mu(t)$ that are δ -correlated in time:

$$\widehat{G}_{\mu\nu}(t-s) = G_{\mu\nu} \delta(t-s) , \quad (3.13a)$$

$$\widehat{W}_{\mu\nu}(t-s) = W_{\mu\nu} \delta(t-s) , \quad (3.13b)$$

with $G_{\mu\nu}$, $W_{\mu\nu}$ time-independent matrices, such that $W = W^\dagger$. From the definition (2.20), one immediately finds:

$$\widehat{\mathcal{B}}_{\mu\nu} = G_{\mu\nu} , \quad \widehat{\mathcal{C}}_{\mu\nu} = 2W_{\mu\nu} , \quad (3.14)$$

⁶⁾ However, when the noise correlations assume the more general exponential behaviour $\widehat{W}_{\mu\nu}(t-s) = W_{\mu\nu} e^{-\lambda_{\mu\nu}|t-s|}$, $\lambda_{\mu\nu} \geq 0$, the corresponding matrix $\widehat{\mathcal{C}}_{ij}$ is no longer automatically positive. In this case, complete positivity needs to be imposed as an additional requirement.

without any approximation, and all considerations about the properties of the generalized dynamics Γ_t presented in the previous subsection can be repeated here. The present case is nevertheless special: because of the correlations (3.13), all higher order terms in the expansion (2.17) identically vanish, so that the evolution equation (2.18) is in this case exact.

4. OBSERVABLES

The evolution in time of the neutral kaons in the medium is fully described by the reduced density matrix $\hat{\rho}(t)$, solution of the master equation (2.18): any physical property of the system can be extracted from it by taking its trace with suitable hermitian operators. Of particular interest are the observables $\hat{\mathcal{O}}_f$ that are associated with the decay of neutral kaons into a final state f , typically 2π , 3π and semileptonic $\pi\ell\nu$ states. In the fixed $\{|K^0\rangle, |\bar{K}^0\rangle\}$ basis, $\hat{\mathcal{O}}_f$ can be represented by a 2×2 matrix, whose entries are expressed in terms of the two independent decay amplitudes in vacuum $\mathcal{A}(K^0 \rightarrow f)$, $\mathcal{A}(\bar{K}^0 \rightarrow f)$:⁷⁾

$$\hat{\mathcal{O}}_f = \begin{bmatrix} |\mathcal{A}(K^0 \rightarrow f)|^2 & [\mathcal{A}(K^0 \rightarrow f)]^* \mathcal{A}(\bar{K}^0 \rightarrow f) \\ \mathcal{A}(K^0 \rightarrow f) [\mathcal{A}(\bar{K}^0 \rightarrow f)]^* & |\mathcal{A}(\bar{K}^0 \rightarrow f)|^2 \end{bmatrix}. \quad (4.1)$$

The corresponding physical observable, directly accessible to the experiment, is given by its mean value:

$$\langle \hat{\mathcal{O}}_f \rangle(t) = \text{Tr} \left[\hat{\mathcal{O}}_f \hat{\rho}(t) \right], \quad (4.2)$$

whose evolution in time is regulated by that of $\hat{\rho}(t)$.

Explicit solutions $\hat{\rho}(t)$ of the master equation (2.18) can be obtained using perturbation theory. As discussed in the previous section, the entries of the coefficient matrix $\hat{\mathcal{C}}_{ij}$ characterizing the dissipative contribution \hat{L} turn out to be of the form W_{ij}/λ , where W_{ij} are the coefficients of the noise correlations, while λ represents their typical inverse decay time. From the analysis at the end of Section 2, it follows that their magnitude can be evaluated to be of order: $|\hat{\mathcal{C}}_{ij}| \simeq 10^{-4} \Delta\Gamma^{(0)}(\Delta\Gamma^{(0)}/\lambda)$, which is small, since by hypothesis $\lambda \gg \Delta\Gamma^{(0)}$. These considerations allow treating the dissipative piece \hat{L} in (2.18) as a perturbation to the contributions of the effective hamiltonian H , which contains the standard Weisskopf-Wigner term $H^{(0)}$ of (2.7), besides the smaller, higher order noise contributions $H^{(1)}$ and $H^{(2)}$.

It is convenient to work in a basis in which H is diagonal: all physical quantities (4.2), being the result of a trace operation, are basis independent. This can be obtained by a similarity transformation analog to the one in (2.5):

$$H = V H_D V^{-1}, \quad (4.3)$$

⁷⁾ In the adopted treatment, kaon decays, being the result of weak interaction processes, are unaffected by the presence of the media; all media influences are encoded in the dynamical equation (2.18).

with

$$V = \begin{bmatrix} p_S & p_L \\ q_S & -q_L \end{bmatrix}, \quad H_D = \begin{bmatrix} \lambda_S & 0 \\ 0 & \lambda_L \end{bmatrix}, \quad (4.4)$$

where, as before, $\lambda_S = m_S - i\gamma_S/2$, $\lambda_L = m_L - i\gamma_L/2$ are the two eigenvalues of H , while p_S, p_L, q_S, q_L define the corresponding eigenvectors $|K_S\rangle, |K_L\rangle$ through definitions similar to those in (2.2).

This change of basis induces the transformation $\hat{\rho} \rightarrow \rho = V^{-1} \hat{\rho} V^{-1\dagger}$ on the density matrix, while the master equation (2.18) becomes:

$$\frac{\partial \rho(t)}{\partial t} = -iH_D \rho(t) + i\rho(t) H_D^\dagger + L[\rho(t)], \quad (4.5)$$

where $L[\rho]$ is again of the form (2.18c), but with a new coefficient matrix \mathcal{C}_{ij} , whose entries are linear combinations of the ones of the original matrix $\hat{\mathcal{C}}_{ij}$ (*cf.* (3.11)).⁸⁾

Similarly, also the observables $\hat{\mathcal{O}}_f$ get transformed, $\hat{\mathcal{O}}_f \rightarrow \mathcal{O}_f = V^\dagger \hat{\mathcal{O}}_f V$, so that the corresponding mean value $\langle \hat{\mathcal{O}}_f \rangle$ remains indeed unchanged:

$$\langle \hat{\mathcal{O}}_f \rangle \equiv \langle \mathcal{O}_f \rangle = \text{Tr}[\rho(t) \mathcal{O}_f]. \quad (4.6)$$

The entries of the transformed matrix \mathcal{O}_f can be conveniently expressed in terms of the following two complex quantities:[17, 18]

$$\lambda_S^f = \frac{q_S}{p_S} \frac{\mathcal{A}(\overline{K^0} \rightarrow f)}{\mathcal{A}(K^0 \rightarrow f)}, \quad \lambda_L^f = \frac{q_L}{p_L} \frac{\mathcal{A}(\overline{K^0} \rightarrow f)}{\mathcal{A}(K^0 \rightarrow f)}, \quad (4.7)$$

as

$$\mathcal{O}_f = |\mathcal{A}(K^0 \rightarrow f)|^2 \begin{bmatrix} |p_S|^2 |1 + \lambda_S^f|^2 & p_S^* p_L (1 + \lambda_S^f)^* (1 - \lambda_L^f) \\ p_S p_L^* (1 + \lambda_S^f) (1 - \lambda_L^f)^* & |p_L|^2 |1 - \lambda_L^f|^2 \end{bmatrix}, \quad (4.8)$$

or alternatively in terms of their reciprocal $\mu_S^f = 1/\lambda_S^f$, $\mu_L^f = 1/\lambda_L^f$, so that

$$\mathcal{O}_f = |\mathcal{A}(\overline{K^0} \rightarrow f)|^2 \begin{bmatrix} |q_S|^2 |1 + \mu_S^f|^2 & -q_S^* q_L (1 + \mu_S^f)^* (1 - \mu_L^f) \\ -q_S q_L^* (1 + \mu_S^f) (1 - \mu_L^f)^* & |q_L|^2 |1 - \mu_L^f|^2 \end{bmatrix}. \quad (4.9)$$

Using standard perturbation theory, solutions of (4.5) can then be easily obtained to any needed order in the small parameters \mathcal{C}_{ij} , while keeping an exact dependence on the quantities parametrizing the effective hamiltonian H , *i.e.* masses m_S, m_L and widths γ_S ,

⁸⁾ Explicitly, one has: $L[\cdot] = V \hat{L}[V \cdot V^\dagger] V^\dagger$; however, since the coefficients $\hat{\mathcal{C}}_{ij}$ in \hat{L} are small, in keeping with the approximation used before, one can neglect CP and CPT violating effects in matter, and set $p_S = p_L = q_S = q_L = 1/\sqrt{2}$ in the above transformation law.

γ_L in matter, and the ratios $r_S = p_S/q_S$, $r_L = p_L/q_L$. While these constants describe familiar hamiltonian behaviour plus CPT and T violating effects in matter [through the combinations θ and $\xi \equiv (|\sigma| - 1)/(|\sigma| + 1)$], the additional piece L in (4.5) is responsible for matter-induced non-standard effects, leading to dissipation and quantum decoherence. In the following we shall focus on the latter, analyzing in detail how the new effects modify the behaviour of selected neutral kaon observables.

5. SINGLE NEUTRAL KAONS

We shall start by discussing the time evolution of observables associated with the propagation in random matter of single, uncorrelated neutral kaons; typically, these can be studied in fixed target experiments (*e.g.* see [30]). As discussed in the previous section, relevant physical quantities are the probability rates that an initial neutral kaon decays at a certain time t into a given final state f :

$$\mathcal{P}_f(K^0; t) = \text{Tr}[\mathcal{O}_f \rho_{K^0}(t)] , \quad \mathcal{P}_f(\overline{K}^0; t) = \text{Tr}[\mathcal{O}_f \rho_{\overline{K}^0}(t)] , \quad (5.1)$$

where $\rho_{K^0}(t)$, $\rho_{\overline{K}^0}(t)$ represent the evolution according to (4.5) of initially pure K^0 , \overline{K}^0 mesons states.

The case of semileptonic final states is of particular interest. The amplitudes for the decay of a K^0 or \overline{K}^0 state into $\pi^- \ell^+ \nu$, $\pi^+ \ell^- \bar{\nu}$ can be parametrized as follows:[31]

$$\mathcal{A}(K^0 \rightarrow \pi^- \ell^+ \nu) = \mathcal{M}(1 - y) , \quad (5.2a)$$

$$\mathcal{A}(\overline{K}^0 \rightarrow \pi^+ \ell^- \bar{\nu}) = \mathcal{M}^*(1 + y^*) , \quad (5.2b)$$

$$\mathcal{A}(K^0 \rightarrow \pi^+ \ell^- \bar{\nu}) = z \mathcal{A}(\overline{K}^0 \rightarrow \pi^+ \ell^- \bar{\nu}) , \quad (5.2c)$$

$$\mathcal{A}(\overline{K}^0 \rightarrow \pi^- \ell^+ \nu) = x \mathcal{A}(K^0 \rightarrow \pi^- \ell^+ \nu) , \quad (5.2d)$$

where \mathcal{M} is a common factor, while the parameters x , z measure violations of the $\Delta S = \Delta Q$ rule and y signals violations of the CPT -symmetry. These quantities are expected to be very small, so that one can consistently neglect terms containing x , y or z when multiplied by other small parameters, coming either from the hamiltonian (*e.g.* θ and ξ) or the dissipative part of the evolution equation (4.5). In particular, this approximation implies that for semileptonic decays: $\lambda_S^{\ell^+} = \lambda_L^{\ell^+} = x$, $\mu_S^{\ell^-} = \mu_L^{\ell^-} = z$.

Then to first order in the dissipative matter effects, the probabilities (5.1) explicitly read:⁹⁾

$$\begin{aligned} \mathcal{P}_{\ell^+}(K^0; t) = \frac{|\mathcal{M}|^2}{2} & \left\{ e^{-\Gamma t} \cos(\Delta m t) \left[\frac{4 \text{Re}(r_S r_L^*)}{|r_S + r_L|^2} e^{-(A-D)\Delta\Gamma t} - 2 \text{Re}(y + 2C) \right] \right. \\ & \left. + e^{-\Gamma t} \sin(\Delta m t) \left[- \frac{4 \text{Im}(r_S r_L^*)}{|r_S + r_L|^2} - 2 \text{Im}(x) + \text{Re}(B) \right] \right\} \end{aligned}$$

⁹⁾ Since $\Delta\Gamma \simeq 2\Delta m$ to a good approximation, in writing (5.3) and (5.4) we have set the ratio $2\Delta m/\Delta\Gamma$ equal to one when multiplying small parameters.

$$\begin{aligned}
& + e^{-\gamma_L t} \left[\frac{2|r_L|^2}{|r_S + r_L|^2} - \mathcal{R}e(x + y - 2C) + D \right] \\
& + e^{-\gamma_{St}} \left[\frac{2|r_S|^2}{|r_S + r_L|^2} + \mathcal{R}e(x - y + 2C) - D \right] \Bigg\} , \tag{5.3a}
\end{aligned}$$

$$\begin{aligned}
\mathcal{P}_{\ell-}(K^0; t) = \frac{|\mathcal{M}|^2}{2} & \left\{ e^{-\Gamma t} \cos(\Delta m t) \left[-\frac{4e^{-(A-D)\Delta\Gamma t}}{|r_S + r_L|^2} - 2\mathcal{R}e(y) \right] \right. \\
& + e^{-\Gamma t} \sin(\Delta m t) \left[2\mathcal{I}m(z + 2C) - \mathcal{R}e(B) \right] \\
& + e^{-\gamma_L t} \left[\frac{2}{|r_S + r_L|^2} + \mathcal{R}e(y - z) - 2\mathcal{I}m(C) + D \right] \\
& \left. + e^{-\gamma_{St}} \left[\frac{2}{|r_S + r_L|^2} + \mathcal{R}e(y + z) + 2\mathcal{I}m(C) - D \right] \right\} , \tag{5.3b}
\end{aligned}$$

where Δm , $\Delta\Gamma$ and Γ are defined as in (2.3) in terms of masses and widths in medium. Matter induced dissipative effects are controlled by the dimensionless parameters:

$$A = \frac{\alpha + a}{\Delta\Gamma} , \quad B = \frac{\alpha - a + 2ib}{\Delta m} , \quad C = \frac{c + i\beta}{\Delta\Gamma} , \quad D = \frac{\gamma}{\Delta\Gamma} . \tag{5.4}$$

The expressions for $\mathcal{P}_{\ell+}(\overline{K}^0; t)$ and $\mathcal{P}_{\ell-}(\overline{K}^0; t)$ can be obtained from (5.3b) and (5.3a) respectively, by changing the sign of y and C , and letting $r_S \rightarrow 1/r_S$, $r_L \rightarrow 1/r_L$, $x \leftrightarrow z$. For a non fluctuating medium, one has $A = B = C = D = 0$ and the expressions in (5.3) reduce to the standard ones, giving the probability of a semileptonic decay for a kaon that has travelled in a slab of material.

The probabilities \mathcal{P}_ℓ are directly accessible to the experiment. Therefore with a suitable set-up, thanks to the different time dependence in the various pieces of (5.3), it is possible to extract information on the parameters (5.4) and therefore on matter induced decoherence effects. Clearly, this task becomes easier in the case of the generalized regeneration discussed in Sect.3.1: in this case, the stochastic medium fluctuations are such that $A = D = B/2$, $C = 0$.

Similar considerations apply to the study of the decay of a single neutral kaon into 2π or 3π final states. For instance, in the just mentioned simplified case, the 2π -decay rate is sensible to the surviving dissipative constant A :

$$\begin{aligned}
\mathcal{R}_{2\pi}(t) \equiv \frac{\text{Tr}[\mathcal{O}_{2\pi} \rho_{K^0}(t)]}{\text{Tr}[\mathcal{O}_{2\pi} \rho_{K^0}(0)]} & = e^{-\gamma_{St}} \left[\frac{|r_S|^2 |1 + \lambda_S^{2\pi}|^2}{|r_S + r_L|^2} - A \right] \\
& + e^{-\gamma_L t} \left[\frac{|r_L|^2 |1 - \lambda_L^{2\pi}|^2}{|r_S + r_L|^2} + A \right] + 2e^{-\Gamma t} |\eta_{2\pi}| \cos(\Delta m t - \phi_{2\pi}) , \tag{5.5}
\end{aligned}$$

where

$$\eta_{2\pi} \equiv |\eta_{2\pi}| e^{i\phi_{2\pi}} = \frac{rsr_L^* (1 + \lambda_S^{2\pi})(1 - \lambda_L^{2\pi})^*}{|r_S + r_L|^2}, \quad (5.6)$$

while $\lambda_S^{2\pi}$, $\lambda_L^{2\pi}$ are the decay parameters defined in (4.7).

More in general, combining the semileptonic and pion decays probabilities in suitable asymmetries one can obtain enough independent observables to treat the case of more general media, for which the parameters in (5.4) are all different from zero.

6. CORRELATED NEUTRAL KAONS

Matter induced dissipative effects can be further studied in experiments at ϕ -factories, using correlated neutral kaons.[19] Indeed, these set-ups behave like quantum interferometers and therefore are particularly suitable for analyzing phenomena leading to loss of quantum coherence.

In a ϕ -factory, correlated kaons are produced from the decay of the ϕ meson. Since this is a spin-1 particle, its decay into two spinless bosons produces an antisymmetric spatial state. In the ϕ rest frame, the two neutral kaons are then seen flying apart with opposite momenta, and in the basis $|K^0\rangle$, $|\overline{K}^0\rangle$, the resulting state can be described by:

$$|\psi_A\rangle = \frac{1}{\sqrt{2}} \left(|K^0, -p\rangle \otimes |\overline{K}^0, p\rangle - |\overline{K}^0, -p\rangle \otimes |K^0, p\rangle \right). \quad (6.1)$$

The corresponding density operator $\hat{\rho}_A = |\psi_A\rangle \langle\psi_A|$ is represented by a 4×4 matrix, since now it describes two kaons. The time evolution of the correlated two kaon system can be expressed in terms of the single meson dynamics Γ_t generated by the equation (2.18). Indeed, once produced in a ϕ -decay, the two kaons evolve independently, so that the density matrix that describes the situation in which the first kaon has evolved up to (proper) time t_1 , while the second one up to (proper) time t_2 is given by:

$$\hat{\rho}_A(t_1, t_2) \equiv \left(\Gamma_{t_1} \otimes \Gamma_{t_2} \right) [\hat{\rho}_A]. \quad (6.2)$$

Correspondingly, one can now study double decay observables, *i.e.* the probability that a kaon decays into the final state f_1 at time t_1 , while the other kaon decays at time t_2 into the final state f_2 :

$$\mathcal{P}(f_1, t_1; f_2, t_2) = \text{Tr} \left[\left(\hat{\mathcal{O}}_{f_1} \otimes \hat{\mathcal{O}}_{f_2} \right) \hat{\rho}_A(t_1, t_2) \right]; \quad (6.3)$$

here, $\hat{\mathcal{O}}_{f_1}$, $\hat{\mathcal{O}}_{f_2}$ are the 2×2 hermitian matrices introduced in (4.1) that describe the decay of a single kaon into the final states f_1 , f_2 , respectively.¹⁰⁾

¹⁰⁾ For the actual computation of the probabilities $\mathcal{P}(f_1, t_1; f_2, t_2)$, it is again convenient to work in the basis in which the effective hamiltonian H is diagonal, and therefore use $\rho_A = [V^{-1} \otimes V^{-1}] \hat{\rho}_A [V^{\dagger-1} \otimes V^{\dagger-1}]$, and \mathcal{O}_f in (4.8), (4.9).

The probability rates in (6.3) are very sensitive to matter induced decoherence effects. This is most strikingly shown by considering correlated decays at equal time $t_1 = t_2 = t$ into the same final state $f_1 = f_2 = f$. In absence of the dissipative term \hat{L} in the evolution equation (2.18), the antisymmetry properties of the initial state $\hat{\rho}_A$ would be preserved by the factorized evolution $\Gamma_t \otimes \Gamma_t$, thus producing a vanishing result for $\mathcal{P}(f, t; f, t)$. The equal time probabilities $\mathcal{P}(f, t; f, t)$ are therefore particularly suited to signal the presence of the dissipative parameters in (5.4). For instance, for f either a 2π or a 3π final state, with associate intrinsic CP parity ζ_f , the probability $\mathcal{P}(f, t; f, t)$ is sensitive to the dissipative parameter D ; explicitly, one finds:

$$\mathcal{P}(f, t; f, t) \propto 2D \left(e^{-\gamma_L t} - e^{-\gamma_S t} \right) \left[(1 + \zeta_f) e^{-\gamma_S t} + (1 - \zeta_f) e^{-\gamma_L t} \right], \quad (6.4)$$

the proportionality constant being a decay amplitude normalization factor dependening on whether $f = 2\pi$ or 3π . On the other hand, in the case of semileptonic final states, it is the parameter a that determines the slope at which the corresponding joint probability approaches zero for small times; indeed, one has:

$$\mathcal{P}(\ell^\pm, t; \ell^\pm, t) \sim a t. \quad (6.5)$$

Although these probabilities together with the more general ones in (6.3) can be measured at a ϕ -factory, much of the experimental analysis performed at these set-ups is devoted to the study of integrated distributions at fixed interval $t = t_1 - t_2$: [32]

$$\Gamma(f_1, f_2; t) \equiv \int_0^\infty dt' \mathcal{P}(f_1, t' + t; f_2, t'), \quad t \geq 0. \quad (6.6)$$

A particularly interesting observable that can be constructed with these integrated probabilities involves 2π final states:

$$\mathcal{A}_{\varepsilon'}(t) = \frac{\Gamma(\pi^+ \pi^-, 2\pi^0; t) - \Gamma(2\pi^0, \pi^+ \pi^-; t)}{\Gamma(\pi^+ \pi^-, 2\pi^0; t) + \Gamma(2\pi^0, \pi^+ \pi^-; t)}; \quad (6.7)$$

it allows determining the ratio ε'/ε , where ε and ε' are the familiar phenomenological constants parametrizing the decay in two pions of the short and long-lived kaons in vacuum. [17, 18] In a medium, the asymmetry $\mathcal{A}_{\varepsilon'}(t)$ gets new contributions, both from the effective hamiltonian H and the dissipative term of (2.18). To lowest order in all small parameters, one explicitly finds:

$$\mathcal{A}_{\varepsilon'}(t) = 3 \operatorname{Re} \left(\frac{\varepsilon'}{\varepsilon} \right) \frac{N_1(t)}{N_3(t)} - 3 \operatorname{Im} \left(\frac{\varepsilon'}{\varepsilon} \right) \frac{N_2(t)}{N_3(t)}, \quad (6.8)$$

where

$$\begin{aligned} N_1(t) &= |\tilde{\varepsilon}|^2 \left(e^{-\gamma_L t} - e^{-\gamma_S t} \right), \\ N_2(t) &= 2 |\tilde{\varepsilon}|^2 e^{-\Gamma t} \sin(\Delta m t), \\ N_3(t) &= e^{-\gamma_L t} (|\tilde{\varepsilon}|^2 + D) + e^{-\gamma_S t} \left(|\tilde{\varepsilon}|^2 - \frac{\gamma_L}{\gamma_S} D \right) - 2 |\tilde{\varepsilon}|^2 e^{-\Gamma t} \cos(\Delta m t), \end{aligned} \quad (6.9)$$

and $\tilde{\varepsilon} = \varepsilon + \varepsilon_L - \varepsilon_L^{(0)}$, with $\varepsilon_L = (r_L - 1)/(r_L + 1)$, $\varepsilon_L^{(0)} = (r_L^{(0)} - 1)/(r_L^{(0)} + 1)$. A careful analysis of the time behaviour of the two contributions in (6.8) would provide a way to estimate the dissipative parameter D , together with real and imaginary part of $\varepsilon'/\tilde{\varepsilon}$. Further, note that in the long time limit the asymmetry (6.8) reduces to:

$$\mathcal{A}_{\varepsilon'}(\tau) \sim 3 \operatorname{Re}\left(\frac{\varepsilon'}{\tilde{\varepsilon}}\right) \frac{|\tilde{\varepsilon}|^2}{|\tilde{\varepsilon}|^2 + D}, \quad (6.10)$$

and not simply to $3\operatorname{Re}(\varepsilon'/\varepsilon)$, as in vacuum. Therefore, even assuming $\varepsilon_L \simeq \varepsilon_L^{(0)}$, a measure of $\mathcal{A}_{\varepsilon'}$ can no longer provide a determination of $\operatorname{Re}(\varepsilon'/\varepsilon)$ unless an estimate on the matter induced dissipative parameter D is independently given.¹¹⁾

7. DISCUSSION

Neutral kaons propagating in a stochastically fluctuating medium can be treated as an open system, *i.e.* as a subsystem immersed in an external environment. Starting from a microscopic hamiltonian with a generic, linear kaon-matter interaction term, a generalized subdynamics for the kaon states has been explicitly derived by averaging over the matter noise. It takes the form of a completely positive quantum dynamical semigroup, where the presence of the medium manifest itself through *i)* the generation of hamiltonian corrections, that modify the familiar Weisskopf-Wigner description of the neutral kaon system, and *ii)* the addition of extra pieces inducing dissipation and loss of quantum coherence.

Some of the hamiltonian contributions have been analyzed before in connection with the so-called regeneration phenomena: they arise because of the coherent interaction of the travelling kaons with the scattering centers in the medium. On the contrary, the remaining hamiltonian pieces and the new dissipative contributions in the kaon evolution equation originate from the stochastic correlations in the medium: as they move in the material, the kaons encounter density fluctuations, whose correlations decay in time very rapidly with respect to the typical time scale of the kaon system, thus inducing irreversibility and decoherence in their dynamics.

Many physical phenomena can give rise to short time correlations in ordinary materials: they have been studied by the so-called femtosecond chemistry.[24] By suitably inserting one of these materials in any standard kaon-physics set-up, one can experimentally study the new, matter induced dissipative effects. Indeed, as discussed in Sect. 5 and 6, one finds that both single and correlated kaon decay observables get modified in a very specific way by the presence of these effects; as a result, they can be probed quite independently from other kaon physics phenomena.

Although here derived in a specific context, the generalized evolution equation (2.18) has wider validity: it has been shown to generate the most general open system dynamics compatible with a semigroup composition law and the requirement of complete

¹¹⁾ A non vanishing D would decrease $\mathcal{A}_{\varepsilon'}$, making $\operatorname{Re}(\varepsilon'/\varepsilon)$ bigger than measured. In the case of string induced dissipative effects, this phenomenon have been discussed in detail in [33]. Similar conclusions have also been mentioned in [25].

positivity.[1-3] As such, it has been recently applied to the description of dissipative effects induced at low energies by the dynamics of fundamental objects (strings and branes) at a very high scale, typically the Planck mass.[34] Although very small, these string induced decoherence effects might be experimentally studied using interferometric devices, like ϕ -factories,[35, 36] and can in principle interfere with the phenomena described here. Notice however that the two situations correspond to totally different experimental conditions. Matter generated phenomena are completely under the experimental control; the effects they induce can easily be isolated from those that might be generated at Planck scale by suitably varying the experimental conditions.

The possibility of choosing and tuning at will the experimental set-up further allows performing interesting tests on the physical consistency of the dynamics generated by (2.18), and in particular on the property of complete positivity.[29] Consider the case in which only one of the two correlated kaons coming from a ϕ -meson decay actually propagates in a stochastic medium, while the other evolve in vacuum. The density matrix that describes this situation at time t is given by $\tilde{\rho}_A(t, t) = (\Gamma_t \otimes \Gamma_t^{(0)})[\hat{\rho}_A]$, where Γ_t is the map generated by the equation (2.18) evolving the kaon in the medium, while $\Gamma_t^{(0)}$, generated by the Weisskopf-Wigner hamiltonian $H^{(0)}$ of (2.7), describes the propagation in vacuum of the second kaon.

As mentioned in Sect.2, any density matrix must be positive; this requirement comes from the physical interpretation of its eigenvalues as probabilities, that thus must be non-negative. In the case of a medium with diagonal correlations as discussed in Sect.3.2, for which the parameters in (3.11) are such that $a = \alpha$, $b = c = \beta = 0$, the four eigenvalues $\lambda_i(t)$, $i = 1, 2, 3, 4$, of the matrix $\tilde{\rho}_A(t, t)$ above can be explicitly computed:

$$\begin{aligned}\lambda_{1,2}(t) &= \frac{\gamma}{\Delta\Gamma} e^{-i\gamma_{S,L}^{(0)}t} \left(e^{-i\gamma_L t} - e^{-i\gamma_S t} \right), \\ \lambda_{3,4}(t) &= \frac{1}{2} \left\{ \phi_1(t) + \phi_2(t) \pm \left[(\phi_1(t) - \phi_2(t))^2 + 4\psi_1(t)\psi_2(t) \right]^{1/2} \right\},\end{aligned}\tag{7.1}$$

where

$$\begin{aligned}\phi_1(t) &= e^{-[\gamma_S^{(0)} + \gamma_L]t} & \psi_1(t) &= e^{-[\Gamma_+ + \Gamma_-^{(0)} + 2\alpha - \gamma]t} \\ \phi_2(t) &= e^{-[\gamma_S + \gamma_L^{(0)}]t} & \psi_2(t) &= e^{-[\Gamma_+^{(0)} + \Gamma_- + 2\alpha - \gamma]t}.\end{aligned}\tag{7.2}$$

Although $\lambda_1(t)$, $\lambda_2(t)$, $\lambda_3(t)$ are manifestly positive for any t , because of the minus sign in front of the square root, one can check that $\lambda_4(t)$ is non-negative only for

$$2\alpha - \gamma \geq 0.\tag{7.3}$$

This is precisely the inequality that in this case guarantees the condition of complete positivity of the dissipative evolution Γ_t (compare with (3.12)); lacking of it would have led to physically inconsistent dynamics.

The situation just described can certainly be realized at a ϕ -factory, so that, at least in principle, the time behaviour of the above eigenvalues can be experimentally studied, and the inequality (7.3) probed. This would allow a direct test of the condition of complete positivity, thus providing direct experimental support for one of the crucial properties characterizing the quantum dynamics of open systems.

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